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Theme Issue: Recreational Mathematics

Note: This electronic version does not include all of the material included in the print version.

From the President...

Don Scheuer
President, PCTM

It is hard to believe that two years of serving as president of PCTM will come to an end in June. I can still remember “taking the gavel” from Ken Lloyd in June of 1998 and looking forward to an exciting and interesting experience working with all of the wonderful people who make up the Pennsylvania Council of Teachers of Mathematics. It has truly been that kind of experience and I want to again thank everyone for giving me the opportunity to have served the organization in this way.

As I look toward the end of my service, I can only hope that we are all diligently working to recruit young members of our profession to get involved in PCTM and to assume leadership roles in the future. I think that anyone who has had this experience will testify that it is probably one of the richest parts of their professional life and that it is a key ingredient in being a better classroom teacher.

Our Annual Conference in Harrisburg was a whopping success and it was personally satisfying to see so many younger teachers taking part in the conference. I was pleasantly surprised at one point when a young man approached me in the hallway and asked if I remembered him. He turned out to be a former geometry student of mine who has gone on to become a math teacher and was attending his first PCTM conference. We chatted at length about his experience and I encouraged him to continue being active and perhaps someday taking a direct part in PCTM activities. I know there are many more like him out there. We have to seek them out.

Next year, the conference moves to Pittsburgh and many people are already hard at work putting the program together. This conference will be the 50th Annual Meeting and a big celebration is in order. Plan now to be a part of this exciting event by attending, speaking, presiding or all of the above! See you there.

Don Scheuer,

President, PCTM

Correction from Winter 2000 issue...

In the last issue of PCTM Magazine, printing problems resulted in several missing equations in Craig Johnson’s article, Computing Distances to the Stars. Readers interested in a fixed version of this article should visit the PCTM Magazine section of PCTM.org, where the electronic version of the Winter 2000 can be downloaded , viewed, and printed in its entirety.

—KD, ed.

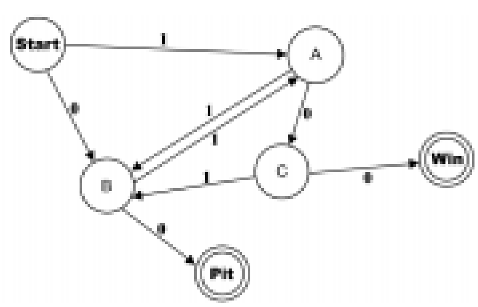
Video Games: Finite State Machines, Finite Automata, Finite Transducers

By James L. Sieber
Shippensburg University

Young people, and "young-at-heart" people, are great video game fans, games in which they proceed through mystery caves, etc., hunting a treasure at the end. An elementary topic from computer science and mathematics that contributes to understanding the logic associated with these games and is accessible at either the high school or college level is finite state machines, FSM, or finite automata, FA. If output is expected, these models are referred to as finite transducers, FT. The study of FSMs is usually found in discrete mathematics and discrete structures courses and is also used in compiler design, language design or theory of computer science courses.

When entering a line of computer code or a complete computer program, the compiler goes through a series of steps and either accepts or rejects it. Similarly a game player goes through a series of inputs and either wins the treasury or fails in the attempt by arriving at an undesirable location such as a bottomless pit. The FTs differs from the FSMs in that an output follows each step just as some games give tools or prizes as the player processes.

A more abstract definition for a FSM, using sets and functions, can be given but a graph theory definition is the clearest. A FSM consists of a labeled directed graph where the vertices are states or locations such as in a cave. The directed edges of the graph are labeled with the possible input characters. In the example below, in order to keep elementary, the only legal inputs will be 0 and 1. One state is identified as the start state and some other states are labeled as final states which can be identified as success in finding the treasure or failure. The remaining states are intermediate states or loops. When an input is received, the player exits the current state along the edge labeled with that input and enters the new state to which the edge points. Consider the following example:



Viewing the graph, only the strings of inputs that contain an odd number of ones followed by two zeros will get the player to the treasure. Strings that contain an even number of ones followed by one zero will arrive at the bottomless pit. Other strings of input will just hang waiting further input until one of the conditions for either the treasure or pit are satisfied. In the above model, once a player arrives at either the treasure or the bottomless pit the game is over. However a student could very easily extend the game by adding further states or intermediate locations and the adjoining edges.

Another very close related computer science topic is grammars with the syntax rules that accompany them. Computer grammars are very similar to the rules of English or other language grammars as they consist of symbols or works and construction rules plus intermediate symbols used in the construction process. A grammar that accepts the same set of strings or sentences as the above FSM is:

Start —0—> B	Start —1—> A	A —1—> B
A —0—> C	B —0—> pit	B —1—> A
C —1—> B	C —0—> win	

where "win" and "pit" are considered final terminal labels that will be replaced by blanks once they received leaving only the string of zeros and ones. If the rules of a current computer language, such as Basic or C, is secured, a student will find that the rules are very similar to the above but consisting of hundreds of such rules and symbols.

Student identification with computer games may help them appreciate the above example and it may motivate them to study more mathematics or computer science. It is suggested that the above model be given to students. To play the model, one student makes up the sequence of zeros and ones, and another follows the model yelling treasure or falling when either of those states are entered. One objective is for the student who makes up the strings and says them to try and discover the tricks to win the treasure. Once mastered, the students should be able to build small similar models. Additional exercises include trying to build the model for one of the popular video games or building a model for some well-known game such as tic-tac-toe. In a solutions of tic-tac-toe, the start state will represent the blank form with the first player having nine choices to locate his mark. Those nine choices will relate to nine edges leading from the start state to one of the nine layouts possible after the first move.

The above example shows both how elementary games relate to advanced concepts and how games can help create an interest in such concepts.

Reference:

Alan Doerr and Kenneth Levasseur, Applied Discrete Structures for Computer Science, SRA, Chicago, 1985.

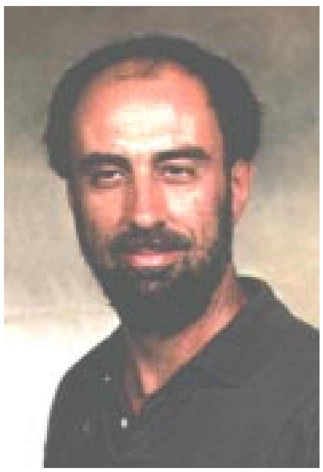
The PCTM Historian

BY DAVID MARCHAND

Bernie Schroeder

Bernie Schroeder began thinking of entering the mathematics education profession as early as fourth grade. If you asked him why this profession appealed to him he would probably tell you that; (1) he wanted to enter a profession that required minimal writing, and (2) he wanted to enter a profession which would allow him to move away from the snow, cold, ice and four-foot long mosquitoes of Central Minnesota.

His formal, higher education began at St. John's University in Collegeville, MN, where he earned a B.A. with a major in physics and a minor in mathematics. He then entered Northwestern University where he earned an M.A.T. Degree in mathematics. After three years of teaching mathematics and physics in secondary school in Chicago and Wisconsin, Bernie returned to graduate school where he earned a M.S. Degree in mathematics from the University of Notre Dame. At this point in time Bernie became an Assistant Professor of Mathematics at The University of Wisconsin-Platteville. While teaching in Platteville (and reaching the rank of Associate Professor of Mathematics), he continued his graduate studies at The University of Wisconsin-Madison. He received his Ph.D. in Mathematics Education from the University of Wisconsin in 1989, having had Tom Romberg as his major professor.



Bernie moved to Millersville, Pennsylvania, in 1989 where he continues to serve the education community as a Professor of Mathematics Education for Millersville University. His duties in that capacity include directorship of the Undergraduate Mathematics Education Program and Coordinator of the Graduate Program in Mathematics Education. At Millersville he teaches three classes each semester including, secondary methods, calculus, and a course in statistics, probability & geometry for elementary education majors. In addition, he teaches graduate courses and offers workshops each summer.

He actually joined the teaching profession because he wanted to be in a profession that was exciting, never dull, and involved interaction with people. He derives great satisfaction when he sees the maturation of students in his methods class over the course of a semester. "Observing a person change from an undergraduate student to a professional, capable of taking charge of a classroom is a very rewarding experience." Other rewards associated with his occupation occur when his former students find employment and when students return to him for counsel and advice.

When asked to complete the sentence, "One thing every teacher should know or do is ...", Bernie offers more than one suggestion. He suggests: (a) "Know your students and tailor your teaching strategies to their learning styles," (b). "Have a sense of humor; don't take yourself too seriously;" and (c) "Don't try to clone yourself; rather, facilitate learning in all students".

Bernie has several publications, including three publications in The PCTM Yearbook, and was a contributing author to Academic Standards for Mathematics, A 1996 publication of the Pennsylvania Department of Education. He has given numerous presentations related to mathematics education at local, state and national levels. He was appointed to PCTM's Ad Hoc Committee on Standards for Algebra and Geometry in 1995 and was elected treasurer of PCTM in 1998. He is currently serving as president-elect of PCSM.

Bernie and his wife, Mary Anne, have four grown children, all raised in Platteville, Wisconsin. All of their children graduated from St. Olaf College in Minnesota, and all majored in mathematics (Bernie says, "because they didn't like to write."). They have three grandchildren who are reported to be "good problem solvers, but are too young to have mastered the basic facts."

Many of us know that Bernie and Mary Anne like to dance, but did you know that the two of them became Life Masters in Duplicate Bridge in October, 1998? Bernie carries a 10-12 handicap in golf, plays once or twice a week, and is constantly "whipped" by their four children during their annual holiday golf timeshare vacation at Hilton Head, SC.

Few PCTM members know that Mary Anne and Bernie retired from the religious folk group Steadfast, soon after they opened for Kathy Troccoli, Michael W. Smith and Boyz 2 Men at Pier 6 in Baltimore (Fall, 1996).

One has to admit that Bernie's profile adds a new dimension to PCTM's set of personality profiles.

BY MARY ANN MATRAS
EAST STROUDSBURG UNIVERSITY

Recreational Math or Practice on Problem Solving Skills?

How do we teach pre-service teachers-of any level-to use, to model and to teach problem solving skills? One way is through the use of interesting recreational math problems of sufficient difficulty that a number of problems solving skills are needed for the solution. A favorite of mine-difficult enough for secondary majors but accessible to elementary education majors-is the Zebra problem. This problem has been around for a long time and has appeared in a number of forms. I remember solving a form of it in junior high school when some of the clues related to type of cigarette smoked!

So, find out who owns the Zebra. The solution is in reverse type under the Happy Endings column on the inside back cover of this issue; just hold the page up to a mirror to read the solution!

Who owns the Zebra?

There are five houses in a line, all of a different color. Each is occupied by a man of different nationality and each man owns a different pet and fruit tree and prefers his own brand of drink.

1. The Italian lives in the red house.
2. The man from Uganda owns an elephant.
3. The coffee is drunk in the green house.
4. The Frenchman drinks tea.
5. The green house is immediately to the right of the white one.
6. The orange grower has a pet hyena.
7. The yellow house has the apple orchard.
8. Milk is drunk in the middle house.
9. The Norwegian lives in the first house on the left.
10. The peach grower lives next to the donkey owner.
11. The lemon grower drinks sherry.
12. The Japanese gentleman grows quinces.
13. The Norwegian lives next to the blue house.
14. Kangaroos eat the next door man's apples.
15. The tea drinker lives immediately to the right of the vodka drinker.
16. The man from Uganda drinks neither milk nor coffee.

PCTM Awards

The following awards were presented at the PCTM banquet, Thursday March 23,2000 at the 49th PCTM Annual Meeting in the Harrisburg Marriott, Harrisburg, PA.

Annalee Henderson Outstanding Student Award
Andrew Aymeloglu, Emmaus High School

Andrew Aymeloglu, son of Barbara and Simeon Aymeloglu was presented the Annalee Henderson Outstanding Student Achievement Award. His teacher Suzanne Moll described Andrew as an outstanding student. Andrew was the top scorer in the Johns Hopkins Talent Search in Pennsylvania while he was in fifth grade and took a precalculus course at Muhlenburg College the summer of eighth grade. He has helped Emmaus High School attain first place in the state in the Pennsylvania Mathematics League Competitions for the past two years. He is a National Merit Semifinalist and an AP Scholar of Distinction. Andrew will be attending Stanford University in the Fall. He will be enrolled in a BA/MA combination in math and physics and will continue on to a Ph.D. program working in the area of math, physics, computer science, and astrophysics.

Outstanding Contribution to PCTM
Ann Massey

Fred Stewart described Ann as a tireless worker who has served PCTM in many different capacities. Ann has been a mathematics specialist in Pitt's Learning Skill Center and taught college mathematics at Pitt and has supervised student teachers. She has taught content and methods courses for elementary education, special education and secondary education. She was the first Membership and Growth Committee Chair for the Pennsylvania Council of Teachers of Mathematics. She was the program chair for the last PCTM meeting in Pittsburgh. She has served on several different committees for the PCTM annual meetings. She has made many presentations at the PCTM conferences. She has been a member of NCTM Leading Mathematics to the 21st Century, which involved writing curriculum for the elementary level.

Distinguished Service to PCTM
Glen Blume

Ann Bacon described Glen as a person who gives unselfishly of his time, sharing his time with students and colleagues. Glen has been involved in major multi-year National Science Foundation projects concerned with the teaching and learning of mathematics in schools, primarily in technology intensive environments. He has been author or coauthor of over 50 publications and has made over 60 presentations to mathematics teachers and mathematics educators throughout the United States. Glen has been active in the mathematics education community at the local, state and national level. He is presently the coeditor of the Pennsylvania Council of Teachers of Mathematics Yearbook and is a member of the Pennsylvania Council of Teachers of Mathematics Executive Board.

Outstanding Contribution to Mathematics Education
Dennis Ebersole

Ned Schillow of Lehigh Carbon Community College presented Dennis Ebersole, Professor of Mathematics Education at Northhampton Community his award. Ned described Dennis as a dynamo, a person who is unfazed by the magnitude of work involved to meet a deadline. Dennis has received seven Eisenhower Grants to in-service K-12 teachers of mathematics on implementing the NCTM Standards. He has received two FIPSE grants in collaboration with Moravian College colleagues to develop new precalculus/calculus courses. He conducted the high school math portion of the Governor's Academy at Temple University. He has done ongoing consulting work for several Lehigh Valley school districts and is currently mentoring twenty teachers in the Bethlehem School District. He is currently one of three co-directors of a Preparing Tomorrow's Teachers to Use Technology Capacity Building grant.

Intermediate Ideas

BY CAROLYN WEINGARD
ABINGTON SCHOOL DISTRICT

Carnival Games

According to the old saying, "It doesn't matter if you win or lose, but how you play the game." However, in some cases, it's how you *make* the game that really matters. The project, called Carnival Games, is an optional, culminating activity at the end of a fraction, chance and probability unit. Students are charged with the task of inventing a carnival game that will show a profit. This activity provides students with the opportunity to analyze games and predict their outcomes, to compare actual results, and to collect, analyze and interpret data. In addition, Carnival Games is highly motivational and great fun!

To do this activity, students need some background with chance and probability. Students need to review the vocabulary for chance events and conduct experiments with spinners and coins. They should have also played games in which players simulate buying tickets to play and win prize money and to calculate profits and losses.

Students play "Box Toss."

Here's how my current class of fourth graders tackled the Carnival Game challenge. To introduce the activity, I started with a discussion of common carnival games. I then divided the class into groups of four. Their goal, as previously stated, was to create a carnival game that showed a profit. Then we determined which classes would attend the "carnival". This year, a first and a second grade class were selected. This led us to another important discussion about age-appropriate games. Two boys offered to survey our guest classes and share their results. The surveys indicated that the children attending our carnival preferred card games, target games and dice games to memory, logic/strategy, or spinner games. The survey also provided an excellent opportunity to make a fraction/decimal/percent chart and bar or circle graphs.

Before the actual construction of the games, a basic ticket price and prize list was determined to make calculating profits and losses a bit easier. This year, the class decided that tickets would be worth \$. 10 apiece. Money was represented by colored strips of construction paper which were valued as follows: Black-\$.50, Red-\$.30, Blue-\$.20, Green-\$.l0. Each child received \$2.00 worth of tickets (20 yellow strips of paper) before entering the carnival. Each game started with \$15.00 worth of colored strips in the "bank". Some groups wanted to use real prizes such as old beanie babies, pencils, or candy. This was a first for me and I was a little concerned about how real prizes would effect the outcome of the project. The class agreed, however, that real prizes could be used if a "money" value were given to these prizes so that they could be considered in the profits or losses of the game.

Then students met to plan their Carnival Games. Several issues were raised within the groups:

How much should we charge for a game?
How big should the prizes be?
Should we charge a high ticket price and reward bigger prizes?
Is this game too hard for first graders?
Will boys want to play the game if there are too many flowers on it? (Response: "Who cares about boys anyway?" Issue resolved.)

A detailed plan, including a list of materials for each game was submitted to me. I conferred with each group and construction began. Please be advised - this activity is not for the faint of heart. Until construction is completed, you may find yourself tripping over boxes, balls, and beanie babies. However, once a carnival date was set and our guests were invited, things moved rather quickly. Students were anxious to complete their games, often staying in for recess or working at home.

Carnival Day 2000

This year's menu of Carnival Games boasted the following games.

<u>Target Complex</u>
Pick a color card and roll a ball down a ramp onto concentric circles each with a point value. If you land on the color you picked, you win a prize. Cost: 2 tickets. Prizes range from \$.10 to \$.50.

<u>Box Toss</u>
Roll a die. Then, to win a big prize, you get 3 chances to toss a ball into a hole with the corresponding number. If you get the ball into another hole, you get a smaller prize. Cost: 2 tickets. Prizes range from \$.10 to \$.50.

<u>Penny Toss</u>
Take a penny and toss it on the game board. If you land on a designated shape, you win a prize. Cost: 2 tickets. Prizes range from \$.10 to \$1.00.

(continued on next page)

Beanie Baby Shoot
Pick five beanie baby cards. Choose the beanie baby you wish to use and try to toss it into a basket. Cost: 2 tickets. Prizes ranged from $.10 to $1.00.
Animal Game
Pick an envelope and answer the question contained in the envelope. If you answer correctly, you spin the spinner. If it lands on your animal, you win a prize. Cost: 2 tickets. Prizes range from $.10 to $.50.
Ball Mania
Put a marble on a small ramp and maneuver it all the way through a maze created by hardened play dough to win a big prize. Smaller prizes can be won along the way. Cost: 2 tickets. Prizes ranged from $.20 to $1.00.

Students play "Penny Toss."

On the first day of the carnival, the visitors from first grade played the games for about 30 minutes. When we calculated our profits and losses, the three games with real prizes accumulated the most tickets, particularly Beanie Baby Shoot. However, they also gave away a lot of prize money for this game and, thus, incurring a significant loss. Although somewhat less popular, the three games without real prizes showed profits of $.40 to $1.00. The groups met to discuss whether to make any revisions to their games. Two of the groups that had games without real prizes agreed that, in order to be competitive, they would need to bring in real prizes, too.

The next day we hosted a second-grade class. After they left, we calculated our profits and losses. Here are the results.

Beanie Baby Shoot was still very popular but, by simply moving their baskets farther away and making it harder to win a prize, they went from a $2.20 loss to a $.50 gain.

"Beanie Baby Shoot" proved to be very popular.

Box Toss lost on both days. This group concluded that the reason their game was so popular was because it was easy to win the big prizes. Unfortunately, they made no attempt to revise their game for the second day.

Penny Toss made a profit on both days, but significantly increased its popularity when real prizes were introduced.

Animal Game was not very popular on the first day, even though it made a slight profit. On the second day, the "owners" stopped asking participants to answer questions before spinning the wheel. They also brought in real prizes. Their second day profit was $1.40.

Ball Mania, although an interesting maze game, lost significantly on both days. This group concluded that the game was just too easy. Even when they attempted to revise their prize structure for the second day, they lost $4.50.

Target Complex was the big winner. This group made a profit on the first day but on the second day their profits soared to $4.10. A spokesperson for the group attributed their success to an increase in advertising. They added some eye-catching signs and verbally recruited customers. Target Complex was the only game that didn't offer real prizes.

The overall consensus of my class was that Carnival Games gave them a new perspective of real carnival games and how they operate. Now that spring is in the air and school fairs are popping up all around us, why not try a carnival of your own? Chances are, it will be a winner.

Reference:

Carnival Games, Everyday Mathematics, Grade 4 - published by Everyday Learning Corporation.

Why I Like Teaching Math in First Grade (or, Confessions of a Right-Brainer)

BY JOANNE KINSEY
WEST BRADFORD ELEMENTARY SCHOOL
DOWNINGTOWN, PA

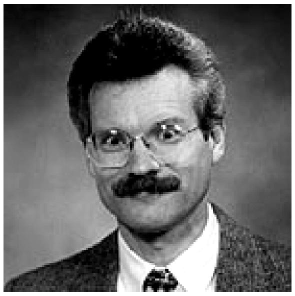
As a college student and as a new teacher I was not particularly thrilled when it came time to teach Math. I fully admit that my areas of expertise are planted firmly on the right side of my head. Reading, writing, plays, artwork were what interested me. Math seemed straightforward and dull, no frills involved. It was to be taught in such a way as to convey the facts and only the facts (my apologies to Joe Friday)

However, over the years, I have found that I can use all those delightful right-brain activities to help me teach Math, especially in first grade. And with all the research on different kinds of intelligence's and learning styles, I need to use every trick in my bag to teach my students. That is why I like teaching Math in first grade.

I can use my musical instruments in teaching patterns and number recognition. We sign songs to help us remember. There are many wonderful stories to make literature connection to any Math topic. Student-made stories can be a great way to assess problem-solving sense and number sentences. Manipulatives can be just plain fun. Games reinforce learning. And the learning is exciting!

In first grade, "you've got it all." I doubt if my college-age child is singing songs to learn about the fine points of accounting or that my high school child is reading stories that connect to geometric theorems. Teaching first grade math encompasses all kinds of teaching styles and methods. It may be hard work, but it is never dull.

(This is part of a series of occasional articles focusing on the reasons for becoming and enjoying being a teacher of mathematics. If you would like to share your story, please e-mail it to Mary Ann Matras at mmatras@po-box.esu.edu. Share your story (and joy!) with others.)



The World of Mathematics: Excursions for Students

BY CRAIG MEROW
GERMANTOWN ACADEMY

A Professor of Puzzles and Paradoxes

The mental health services in France are in a state of chaos. While they have eleven modern insane asylums, no one can tell the patients from the doctors! Inspector Craig of Scotland Yard is called in to investigate.

Upon arriving in France, the inspector collects the following facts:

1. Everyone living in the asylums is either a doctor or a patient.

2. Each inhabitant of the asylums is either completely sane, with totally accurate perceptions, or completely insane, with completely inaccurate beliefs.

3. All residents of the asylums report their beliefs truthfully.

Armed with this information Inspector Craig begins his evaluation of the condition of the asylums:

In the first asylum Craig visited, he spoke separately to two inhabitants whose last names were Jones and Smith.

"Tell me," Craig asked Jones, "what do you know about Mr. Smith?"

"You should call him Doctor Smith," replied Jones. "He is a doctor on our staff."

Sometime later, Craig met Smith and asked, "What do you know about Jones? Is he a patient or a doctor?"

"He is a patient," replied Smith.

The inspector mulled over the situation for a while and then realized that there was indeed something wrong with the asylum: either one of the doctors was insane, hence shouldn't be working there, or, worse still, one of the patients was sane and shouldn't be there at all (Smullyan 1982, 29-30).

Inspector Craig is just one of the colorful characters in Raymond Smullyan's book, The Lady or the Tiger (Smullyan 1982). His exploits in the Asylum of Doctor Tarr and Professor Fether, on the Isle of Dreams and the Island of Questioners, and in Transylvania, lead the reader deep into the world of mathematical logic.

Smullyan has the ability to address serious mathematical principles with fanciful puzzles. As one of his students reported:

He sort of strings you along, and then you find yourself at the heart of Godel's Theorem. You don't notice that you are doing what some people would consider "hard stuff". It all seems very easy. You don't realize you are learning something profound (Mothner 1985, 304).

Many who have found mathematics tedious, frustrating, and at times incomprehensible, are attracted to Smullyan's books. A mathematical colleague, whose son was enjoying one of Smullyan's puzzle books warned the author not to tell the boy that in solving the puzzles he was doing real mathematics. "... he hates math!" the father said. "If he had any idea that this was really math, he would stop reading the book immediately (Smullyan 1982, vii)!"

Raymond Smullyan is a tall, slim man, with a long white beard, a playful smile, and a penchant for pound cake and leisure. For many years he lived with his wife, Blanche, in the tiny New York village of Elka Park. He commuted three hours to Manhattan twice a week to teach his classes at the City University of New York. He spent the remainder of the week at a huge table in his living room, playing with pencil and paper, and the characters who star in his logical puzzles and paradoxes.

Smullyan was not always captivated with logical puzzles. As a youngster he was a musical prodigy, winning the gold medal in a New York City piano competition. Later, he became interested in astronomy and built his own telescope. For a time he worked as a professional magician with the stage name, Five-Ace Merrill. He drifted in and out of college, studying music, philosophy, and mathematics. He completed his B.A. at the University of Chicago when he was thirty-five years old, and went on to earn a Ph.D. in mathematics at Princeton University (Mothner 1985).

While Dr. Smullyan is now a respected emeritus professor of philosophy and mathematics and the author of two technical books on mathematical logic, his sense of fun is still alive and well. With his quick wit, puzzles and paradoxes, and a baffling magic trick or two, he is always prepared to amuse a child or introduce a student to the world of mathematical logic.

Exercises

1. How did Inspector Craig know that the first asylum contained a sane patient or an insane doctor?

2. Read one of the Smullyan puzzle books listed in REFERENCES. Present one of his puzzles to your class.

3. A significant amount of mathematics can be learned by reading "recreational mathematics". The books by Martin Gardner are particularly recommended. Mr. Gardner is, without a doubt, the premier recreational mathematics writer in the United States. For over twenty years he wrote the mathematical games column for Scientific American and has published several books of mathematical puzzles. A few of his works are listed in REFERENCES. Investigate what some of these volumes have to offer.

4. Raymond Smullyan has been described by Peter Denning as "the Lewis Carroll of our times." Lewis Carroll was the pen name of Charles L. Dodgson, the author of Alice's Adventures in Wonderland. Dodgson was a mathematics teacher who wrote the Alice stories as a hobby. They are filled with examples of deliberately faulty logic. Read this children's classic with an eye for faulty logic.

5. Lewis Carroll also wrote books of mathematical problems and puzzles. Try the following problem from Pillow Problems and A Tangled Tale (Carroll 1958, 96).

There are 5 sacks, of which Nos. 1, 2, weigh 12 lbs.; Nos. 2, 3, 13 1/2 lbs.; Nos. 3,4 11 1/2 lbs.; Nos. 4, 5, 8 lbs.; Nos. 1, 3, 5, 16 lbs. Required the weight of each sack.

6. Visit the Raymond Smullyan puzzle site, http://www.cut-the-knot.com, and enjoy!

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For the Teacher

1.If Jones is sane then Smith is a doctor. If Smith is insane, then he is an insane doctor. If Smith is sane then Jones is a patient, a sane patient.

Now if Jones is insane then Smith is a patient. If Smith is sane, then he is a sane patient. If Smith is insane, Jones is a doctor, an insane doctor.

5.5 1/2, 6 1/2, 7, 4 1/2, 3 1/2.

Dates and Locations of Future PCTM Meetings

2001:	Pittsburgh, Marriott and Holiday Inn Greentree, March 15-17, 2001. General chair: Ron Harrell. Hosted by MCWP, AMTONP, and LHMA.
2002:	Philadelphia, March 14-16, 2002. General chair: Rosemary Fogarty. Hosted by ATMOPAV and BCCTM.

Elementary Endeavors

BY MARY ANN STARKEY
WEST BRADFORD ELEMENTARY

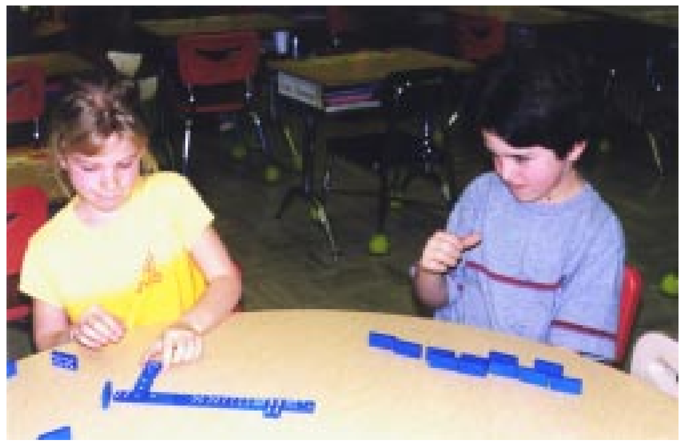
Game Time!

The game sign is up! Students are eager to complete the required assignment to get to the games. What they don't know is that these games require math skills in a fun format.

Dominoes is a very old game, but students still delight in spilling the dominoes out of the bag and challenging a classmate. Matching the number of dots, turning the doubles sideways and trying to outplay the opponent are all math-related concepts. Of course other dominoes have been created such as telling time and money dominoes. The game of dominoes must be back in vogue, because I have had several parents ask for domino rules this year. It's a great family game.

Mancala is another favorite game that requires logical thought and problem solving skills *(see photo on front cover)*. My students begin play by placing three stones in each pit. It's interesting to watch the students determine how to get that last stone in their own bin thus giving them another turn. We also play the rule of capture-that is if the player's last stone falls into an empty pit he can capture all the stones in his opponents opposing pit. In this game the first person who gets rid of all of his stones is not necessarily the winner. The winner is the person with the greatest number of stones. You should have heard the surprise "Oh" when the stones were counted up! Playing mancala requires logical thought, concentration and quick reactions. This game has become so popular that I have had to create additional games using egg cartons and lima beans!

The current favorite card game is pyramid. The cards used are Ace and the numbered cards. All face cards are removed. Two students create a pyramid of cards by placing one at the top and two cards overlapping the top card in the second row, three cards overlapping the two cards in row three continuing down until there are six rows of cards. Only the bottom row of cards is completely exposed. The remainder cards are placed in a pile to be drawn upon. The object of the game is to make combinations of 10 using one, two or three cards. The playable cards are the ones that are totally exposed. When a ten combination cannot be reached a card is drawn from the extra pile. This game can be played by one or two students. (I found a great source for used playing cards by writing to a New Jersey casino. Of course these cards have a small hole in the middle but that doesn't seem to bother anyone!) Logical thought, problem solving and addition facts of ten are the underlying math principles at work here.



Daniel challenges Kate to a game of Dominoes.

Building blocks are a favorite choice for primary students. Blocks hold students' fascination for many reasons. They are solid. Kids can handle them without worrying about breaking something. They have different attributes-some stack, some roll. Making a three-dimensional structure takes concentration, a steady hand and skill.

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Structures can also be made out of gummy bears and toothpicks! During our geometry unit my students made gummy bear/toothpick pyramids. With a handful of gummy bears and toothpicks my students began to create other structures. Soon cubes, triangles and a host of other structures were seen all around the room. The gummy bears and toothpicks had been transformed into a 3D art form.

Some students have a difficult time seeing spatial relationships. Making puzzle pieces fit is a great activity. Students must twist and turn pieces in hopes that they will fill the empty space. When completed the student is rewarded with a completed puzzle. Primary students will spend quite some time analyzing a puzzle piece to determine its place. Puzzles come in all different formats - 3D puzzles, tiles, create-a-picture, and the traditional puzzle. The surprise cry of, "Look I did it!" is heard all over the room when a puzzle is solved. It's not unusual for a student to immediately spill the completed puzzle just to see if he can solve it again!

Pyramid is a favorite with Michael and Ashley.

Creating a picture using pattern blocks, magnetic tiles or construction paper are all fun activities for the primary grade student. Matching and turning, pieces are placed in position for a specific reason. The logic is within the child's mind. Sometimes a child will show a completed picture and describe it, other times the child is delighted within his own creativity. The child is not aware that he is doing some higher level thinking skills- he is just having fun.

Games, puzzles and pictures in math class? You bet! Lots of learning is taking place: logical thought, problem solving, spatial relationships, as well as pro social skills are being developed here. PCTM

Submissions Solicited For PCTM Magazine

You are invited to submit articles for consideration for publication in the PCTM Magazine. This publication, which is published and sent to the membership of PCTM three times each year, provides an excellent opportunity for you to share your ideas with the ever-growing number of colleagues dedicated to improving mathematics education in Pennsylvania.

Any topic of interest to teachers of mathematics is suitable subject material. Submissions intended for consideration for the next issue must be received no later than **September 7, 2000**. Submission may be made by mailing a 3.5" disk (plain text format, either Mac or IBM) and including a printed copy of your article. Any graphics and tables should be camera-ready; or better, they can be submitted as separate graphics (TIF, PICT or BMP formats) on disk. Be sure to include an e-mail address if possible and home and work phone numbers. Send your submission to:

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The Game of SET in Teaching Mathematics

By Patricia J. Fogle, PhD., D.O.
Berea, KY

Problem solving and critical thinking are two of the major goals in the teaching of mathematics. As part of the approach to problem solving and critical thinking, students are expected to master skills in the interpretation of the written or spoken word, and in interpretation of diagrammatic representations of these words. Given a set of rules related to the problem at hand, students are then expected to find an appropriate solution to the problem.

In attempts to facilitate student learning, teachers for years have used manipulatives in the classroom, with various goals in mind. One such group of manipulatives is the attribute blocks, which usually consist of objects of various shapes, colors, and sizes. Another is the game, *SET: The Family Game of Visual Perception*, which teaches students about similarities and differences, and compels the student to recognize many different kinds of patterns.

A SET is defined as three cards that either share a common attribute or are totally different for that attribute. That final card which forms the match for any two cards must conform to the pattern for all attributes on the cards. Each card has four attributes - shape (oval, diamond, or squiggle), shade (open, shaded, or solid), color (red, green, or purple), and number (one, two, or three). An example of a SET will clarify what we mean. [Ed. note: in the figures in this article, the letters, R, G, and P have been inserted to denote the colors red, green and purple; the actual cards are in color.]

Suppose that in the actual play of the game one wants to determine the third card which will complete the SET started by the two cards in the top row of Figure 1.

One should ask, "Are they the same, or are they different, for shape, shade, color, and number?" The completed SET is shown in the bottom row of Figure 1.

Consideration of each attribute can be done in any order, as long as all four attributes are checked. In this example the pattern followed that of similarity or difference for *each* attribute, and this pattern is established as soon as the second card is chosen. One can now see that for any two cards there will always be only one card that will complete the SET.

In actual play, twelve cards are laid face up so that all players can see them. (See Table 1 for a "gold mine" of SETs.) The first person to see a SET calls, "SET." That stops the action until that person has pointed out his selection of cards to the others, and if correct those cards come off the board and are replaced by another three cards. Play resumes and continues until all cards have been dealt, and all possible SETs have been removed. The player who has found the most SETs is the winner.

Several tips are helpful for teachers attempting to introduce their students to SET in their classrooms. These are especially applicable when dealing with young children (4-6 year olds), those with mental/behavioral handicaps, and students who are color-blind or have hearing difficulties. This set of tips is available by writing to Patricia J. Fogle, 31 Elm St., Elm Manor #2, Berea, KY 40403. Also, several tables of activities are included in this article which might be helpful in stimulating their interest.

Once students have become proficient in recognizing various SET patterns, they can use the game to address questions from various mathematical disciplines. A number of questions have been posed and discussed in the article in *Mathematics Teacher*, Dec. 1999, (*Developing Mathematical Reasoning Using Attribute Games*). Several questions from that article can begin to be addressed by developing a magic square of SETs. Rules for developing this magic square are described in "SET and Matrix Algebra." One of these questions is, "Prove that five cards that have two common attribute values must include a SET." This question then might be addressed using a magic square of cards that are all red and all one, from which any five cards are then removed. The four remaining cards may contain no SETs or one SET. The addition of one more card (total of five cards on the square) now allows one or two SETs. Another way of addressing the question uses concepts from set theory. If one starts with a SET and then adds one additional card no new SET can be formed. If one now takes that fourth card and unites it with any one of the three original cards one can then create one new pattern by selecting the appropriate fifth card. One now has two SETs that share a common card, i.e., an intersection of two SETs. If one had, on the other hand, randomly added any other fifth card no new SET would have been possible. In the first instance one has a union of two SETs with the common element creating the intersection of both. In the second instance the two additional cards are considered disjoint from the original SET. If one now continues with the two united SETs one can create four SETs maximum in nine cards total if one uses the intersecting card to create "spokes in a wheel", and if the spokes themselves contain no additional SETs. Consider card #1 as the intersecting card. Then cards 1-2-3, 1-4-5, 1-6-7, and 1-8-9 would constitute your SETs. If, on the other hand, one had used the magic square, laid the original SET in any straight line on the board and placed the fourth card in any of the remaining positions, one could achieve 12 SETs maximum. Each card is the intersection of four SETs and all SETs are maximally interconnected. It should be noted that any three cards that do not make a SET in themselves can define 12 interconnected SETs using the magic square concept.

Another question from the article is, "Prove that among any group of seven cards, exactly four sets cannot exist." This can be addressed by removing any two cards from a completed magic square. The removal of any first card results in a loss of four SETs from the original twelve. The removal of the second card results in a loss of seven SETs, or five SETs remaining. There can be

(continued on page 21)

(continued from page 18)

zero to three SETs in a random group of seven cards, but if seven cards contain four SETs, there must be a fifth SET also. How many SETs might be present in six cards? (Answer: three). Any magic square of SETs will give the same answer. *Does anyone have a mathematical formula to verify what we can visually and logically determine?*

Another question addressed by the magic square relates to the types of patterns found in each category of SETs. For example, what types of SETs would be formed if the second and third cards chosen to form the magic square differed in four attributes from the first card? If the original card selected (rectangle) is used with the additional two cards (ovals) to create the magic square seen in Table 2, the board contains three SETs with one difference, three SETs with three differences, and six SETs with four differences. What if the two additional cards (surrounded by ovals) had differed in one attribute from the original card (surrounded by the rectangle)? What if one additional card had differed in one characteristic, and one had differed in three characteristics? And are the types of patterns seen independent of the two cards selected, or also reflective of the number of differences between the second and third cards?

What is the maximum number of SETs possible in 10, 11, 12, 18, and 27 cards? Is there a mathematical formula that will predict these answers? It is interesting that if one assigns each card a number and then systematically looks at potential combinations in twelve cards, one arrives at the number "17". If one starts with the presumption that the first card can be chosen in twelve ways, the second card in eleven ways, and the third card is then fixed, the answer would be "22", i.e. 12x11x1/6. In reality, there can only be 14 SETs maximum in 12 cards! What gives? And what then about 18 cards? My answer is 24, the equivalent of two magic square totals. If you try to break into either square, you will lose numbers. On the other hand, 27 cards can yield 117 SETs. Here the calculation 27x26x1/6 = 117 matches the number calculated from three interconnected magic squares. You can generate three interconnected magic squares by completing the first square, then placing one additional card on a second square. Generate the other cards by considering the squares as a three-dimensional tic-tac-toe arrangement, and "two cards in a row determine the third card". Each layer of magic square contains 12 SETs and each card in the first tier has nine connections with the second and third tiers - a total of 3x12 and 9x9 - or 117!

In an entire deck of 81 cards it is possible to have nine distinct magic squares at any given time. Once these magic squares are formed, is it possible to arrange each magic square on the side of a 3x3 Rubics Cube in such fashion that two or more new magic squares can be generated out of the 54 cards available? (One new magic square is easy!)

What about using SET to look at concepts from statistics? Two areas that are addressed in the "SET and Statistics" are small population vs. large population statistics, and conditional statistics. There is a tie-in with the questions posed in the *Mathematics Teacher* article regarding probabilities of finding various types of SETs. As mentioned there, probabilities are related to having a whole deck, and everything changes after the first three cards are withdrawn. And in regard to probabilities calculated in Table 1 in that article the question can also be addressed by your visual learners by taking all the red cards from a deck, sorting them, and then visually determining how many cards can be picked for a second card for a particular number of differences. One example is given in Table 3. All types of categories except that of four differences can be considered using this table. In the category of four differences the hatched cards would be on the same positions in the green and purple cards (a total of sixteen cards), for a total of 81x16x1/6 SETs.

And the last question (for now) - what is the maximum number of cards one can have which does not contain a SET? This question has been asked for years, and I am not aware of a mathematical proof that the number is 21 (previously thought to be 20). Several approaches might be tried to answer this question, and just recently a computer-generated group of 21 was determined. We know that two cards will make a SET, and that four of nine in a magic square may not contain a SET. And we know that nine cards can be found in one of the smaller decks (red) which do not contain a SET. What is the next number in this sequence of 2, 4, 9...? Duplicating these same cards in the purple deck gives us 18 cards immediately. But invariably every green card has formed a SET with one red and one purple card. A more fruitful approach involves selecting the nine original cards from a group of 27 sharing a common attribute (e.g., all three), then selecting two from a second group (all two), then eliminating all ones which formed a SET with these eleven cards, and from any remaining cards trying to get nine cards which did not contain a SET. Now, is it possible to go back to the twos and select three cards and try the same approach? If anyone does come up with 21 non-SET cards or with a mathematical proof for the global maximum of non-SET cards please let us know.

In closing, I'd like to share some practicality of seeing the interrelationships of the various groupings. In math and science one draws conclusions from data based on the statistical significance of comparisons of two or more groups. When two groups can be compared with their multiplicity of characteristics, the validity of any conclusions is enhanced when more variables are taken into account. For example, in biology as one progresses from genus to species to phylum characteristics become more defined within each group. In chemistry, the periodic table allows us to compare chemical reactions based on similar electronic structures. In medicine, one could sort our population into 15 or 16 categories based on exercise, smoking, drinking/drugs, and dietary choices - using low/none, moderate, and excessive descriptors. In sociology one could sort groups by parenting, family income, education level, type of community in which raised as reflectors of a person's current status. Answers to questions directed to particular groups then become more meaningful as one narrows down the category into which he/she fits. Hopefully, learning the intricacies of SET will lead to new endeavors in this wide, wide world!

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Happy Endings

Who Wants to be a Millionaire?

In 1742, Christian Goldbach wrote a letter to Leonhard Euler which is probably now one of the most famous letters in mathematical history. In it, he wrote the following conjecture:

Any even number greater than two can be expressed as the sum of two primes.

Now well known as *Goldbach's Conjecture*, this over 250-year-old statement still awaits a proof.

Just this Spring, the British company Faber and Faber published a new book of fiction by Apostolos Doxiadis entitled *Uncle Petros & Goldbach's Conjecture*. Described by John Nash as, "a fascinating picture of how a mathematician could fall into a mental trap by devoting his efforts to a too difficult problem," it should be great reading for teachers and students of mathematics. In connection with its release, the company has offered a one million dollar prize to any person who can prove Goldbach's Conjecture by March 15, 2002.

Details of the one million dollar challenge are available on the Faber and Faber website at www.faber.co.uk. Informaton on the book, which recently retailed at Barnes and Noble for $23.95, is available on Doxiadis' website at www.apostolosdoxiadis.com.

Now, who wants to be a millionaire?

Solution to College Math Corner: The Japanese man owns the zebra.

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