Piecing Together Piecewise Functions

Piecewise functions, typically introduced in Algebra 2, help set the foundation for deeper explorations of the graphs of various functions in pre-calculus and for computing limits in calculus. Therefore, it is important that students gain a solid understanding of piecewise functions before entering either of these branches of mathematics. I began to ask myself how I could present piecewise functions in such a way that my students would gain more than a superficial understanding, allowing them to discover their relevance. My answer came through a guided discovery activity involving connections across the curriculum and a peer project which required problem solving and reasoning skills.

Guided Discovery Activity:

Since algebra 2 students have had significant practice with linear functions, starting with these activities would be best. Pass out the Piecing Together Piecewise Functions - Guided Discovery Activity worksheet (see below) along with rulers and transparency paper. You may guide students through numbers one and two; however, it may be more beneficial to allow them to work through the first two questions with a peer and then come together as a class to discuss questions three and four.

Discussion

Allowing students to share their findings in question three and their definition in question four will help you determine if it would be best to practice graphing more piecewise functions involving linear equations before doing the peer project.

Provide students the opportunity to build on one another's observations and answer each other's questions. This will allow them to use their mathematical language and enhance their communication skills - an NCTM standard (NCTM, 2000). Many students will notice that when both equations are put on one graph, the second equation begins where the first one ended. This makes a continuous function. (The different-colored pencils help students recognize this aspect of the graph.) At this point ask your students, "Will this always be the case? Justify your answer." Once students are confident with their answer and justification, look at an example in which this is not the case. After discussing question three, reiterate how this problem would look when written as a piecewise function in functional notation. Then allow students to share their definitions written for question four. It may be helpful to list key ideas on the board and use those to create a class definition. It is important to stress that different parts of the domain have different roles.

Peer Project:

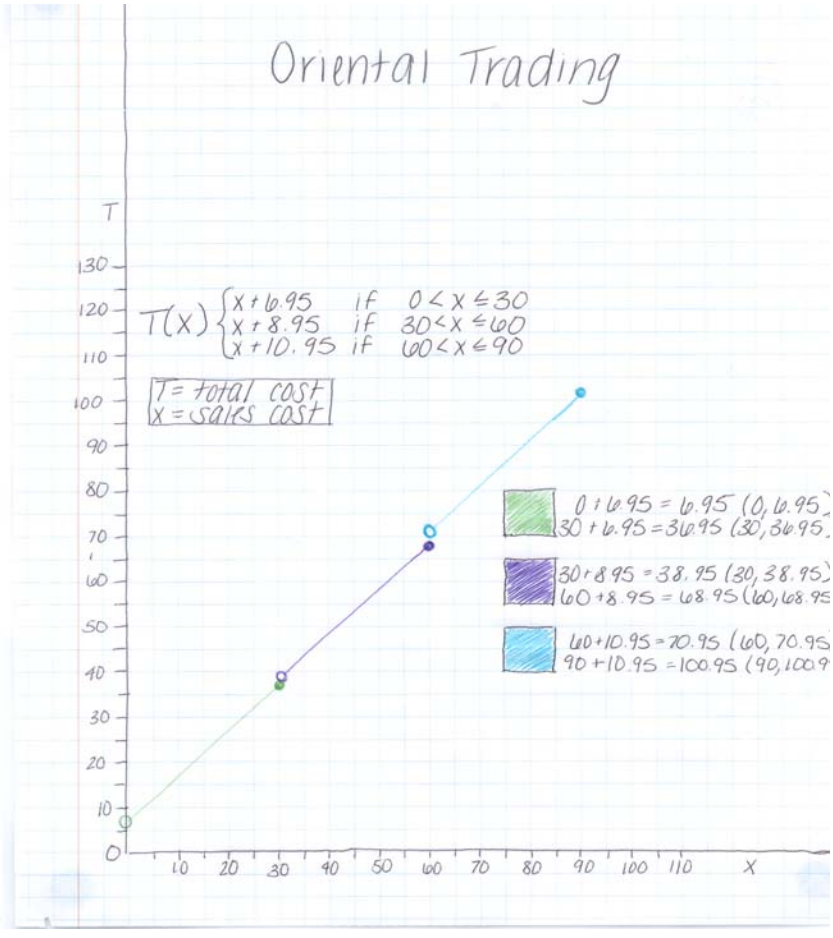
Provide each team of students (I would recommend keeping the teams to two or three students) with a magazine containing shipping costs for a particular company. This project will be more fun for students if the magazines have items that pertain to their interests (sports equipment,

prom party supplies, etc.). Allow students to flip through the magazine for a few moments, and then ask them to find the page that contains the shipping costs. Their task is to write and graph the piecewise function for total cost (cost of order + cost of shipping) for standard shipping on all orders up to and including $90.00. You may need to change this number depending on the shipping cost intervals of the magazine your students are using and how many equations you would like them to include in their function. You may also choose to have them compare shipping costs for different areas of the United States. Before students begin, make sure they understand that they are creating a function for any possible order. Since the order cost will vary from person to person they must use a variable.

Below is a portion of the shipping cost standards for Oriental Trading Company that was used by a team in my classroom for this particular project.

Order Total	**Standard (7-10 business days)**	**Express (3-5 business days)**
Up to $30.00	$6.95	$14.95
$30.01 to $60.00	$8.95	$16.95
$60.01 to$90.00	$10.95	$18.95

Once students have written their piecewise function, you may want to check to make sure it is correct before they take the time to graph it. [Mike, I would enlarge the graphic below when you lay it out. – DK]



Upon conclusion of graphing the function, allow students to share their graphs and findings with their classmates. Here are some questions you may want to pose to help generate a discussion:

- Why do we not include zero when calculating total cost for orders up to $30.00?
- Why do you suppose that express shipping is often less than twice the amount of standard shipping?
- How would your function change if shipping costs for orders greater than $60.00 were 15% of the total cost?
- When would a 15% shipping cost be to the buyer's advantage?

Conclusion:

Graphing piecewise functions and understanding their derivation is only the beginning of obtaining a firm foundation. It is also important that students are able to recognize and write piecewise functions for given scenarios. Here is one example:

> Suppose you plan to buy many blank compact disks. You check price lists and find out that if you buy 100 CDs or fewer you pay $0.74 each. However if you buy between 100 and 300 CDs the price drops to $0.69 each for the second hundred. Write a function that describes the cost *c* of *n* number of CDs purchased.

In my experience, students find writing the function more challenging than graphing the function, thus this aspect of the lesson takes longer. Choosing scenarios which are realistic to them, however, will make the lesson relevant and fun!

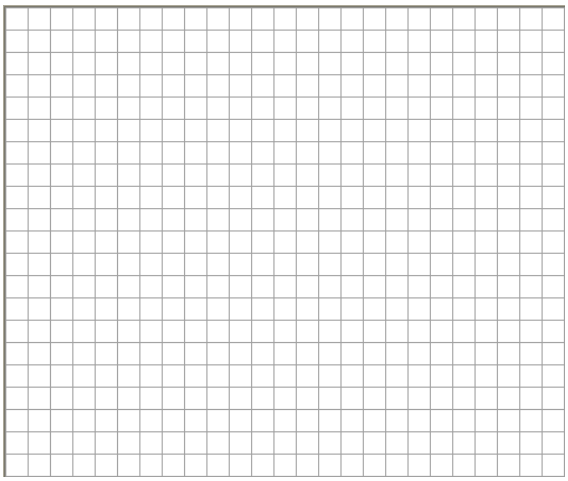
References:

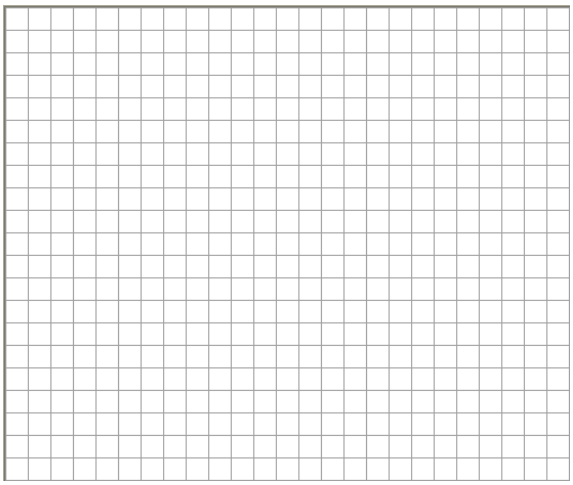
National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Orientaltrading.com

Piecing Together Piecewise Functions - Guided Discovery Activity

1) Given the following linear equations, represent solutions graphically on two separate coordinate planes. $y = 3x$ and $y = -2x+20$



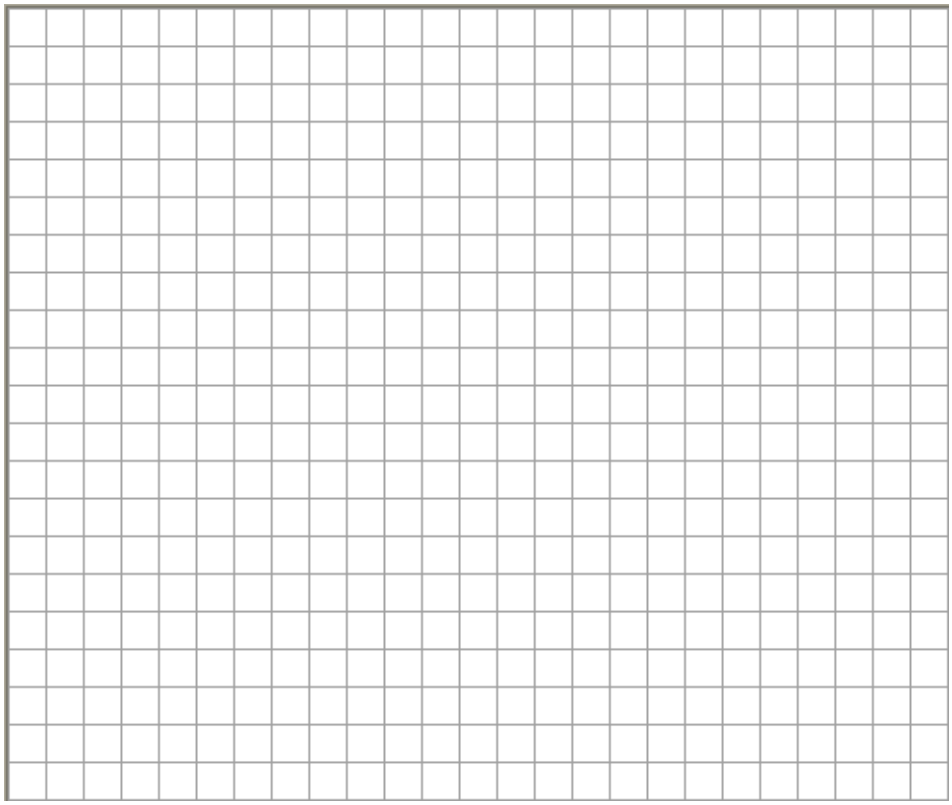


2) Using the coordinate plane below, place a piece of transparent paper on top and graph the solution of the first equation according to the given domain.

$y = 3x$ where $-1 \leq x < 4$

Now place a second piece of transparent paper on top and using a different colored pencil, graph the solution of the second equation according to the given domain.

$y = -2x+20$ where $4 \leq x \leq 6$



3) What are some similarities between your first two graphs and the third one? What are some differences?

Written in functional notation, number two would look like the following:

$$f(x) = \begin{cases} 3x & -1 \le x < 4 \\ -2x + 20 & 4 \le x \le 6 \end{cases}$$

4) This is known as a piecewise function. Using the third graph and the discussion we just had, write your own definition for a piecewise function.